

# Evaluating Facilities Layout Designs using Fuzzy - MCDM method

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**Abstract** - The relationships among the industrial facilities, space available for those facilities and cost involved are the important factors in determining the proper selection of a Facility Layout Design (FLD). Generally, a FLD problem is taken to be an unstructured decision making problem. The real-world fuzziness associated with various factors determining the FLDs pose real difficulties in developing and using ready made models for layout design. Crisp data is always not fully useful but sometimes becomes a hindrance in correctly assessing an industrial scenario. Hence, it is always better to make the situation as structured as possible. In this paper there is an attempt to propose a FLD selection algorithm that is based on a combination of hierarchical structure analysis and Fuzzy Set Theory. The aggregate linguistic appropriation about each 'Selection Criteria Weight' (SCW)s and the assessment regarding the suitability of the given FLDs versus various selection criteria by the experts (both Subjective & Objective criteria involved) are considered to obtain Fuzzy Facilities Layout Index(FFLI). Then FFLI ratings are ranked to select the best suited FLD. This robust algorithm enables us to make a fairly good selection from amongst many alternative FLDs.

**Key Words:** Fuzzy-MCDM, Fuzzy-AHP, Facilities Layout Selection, FFLI, Fuzzy Linguistic Variables (FLVs)

## Introduction

Modern manufacturing systems are different from the traditional set ups both in terms of quantity and quality of output. With JIT concepts being adopted, CIM organisations replacing conventional system, manufacturing arena has seen a 'revolution' in the way the 'business' is looked at. Over the past two decades manufacturing has become more complex due to global competition, great varieties and rapid advances in technology. The development of Flexible Manufacturing Systems (FMS) is of interest from both the economic and technological points of view. An FMS is a series of computer controlled NC machine tools that can randomly process a group of parts, having an integrated material handling system and central computer control to dynamically balance resource utilization, so that the system can adopt automatically to changes in parts production, mixes and levels of output. Today's manufacturing environment is strengthened with CNC, DNC and FMS. The emergence of the new areas such as cognitive science are definitely aimed at solving various industry related problems. \*

The evolution of FMS, CIM (Computer Integrated Manufacturing) has a great potential for increasing flexibility in manufacturing. The rapid emergence of areas in technology such as Robotics, Artificial Intelligence, Knowledge based systems are definitely aimed at solving various industry related day-to-day problems with little human interventio. The basis of competition has undergone a sea change but ensuring both cost effectiveness and customization in manufacturing. FMS has been a focal point in manufacturing related research since

early 1970s. Computer integration and flexibility of the system are the two critical factors of an FMS (Nagarur, 1992). Flexibility is the ability of a manufacturing system to cope with changing circumstances (Buzacott and Mandelbaum (1985)). Flexibility ensures that manufacturing can be both cost effective and be customized at the same time (Gupta and Goyal, 1989). A higher level of flexibility will enable the manufacturing firm to provide faster response to market changes, while maintaining high product quality standards. The problem of FMS operation is also divided into many interconnected sub-problems, each of which can be solved independently. Flexible Layout Design is the need of the hour to effectively save investment up to the tune of 20%( Tompkins & White 1984). Hence in the present context an attempt is made at selection of a proper FL in the rapidly changing modern industrial environment. Current global economic recession demands a total relook into the manufacturing sector. 'Cost cutting' has become the most thought out subject. With this background there is a need to optimize the FL design and make the right choice of it.

## Prior Art

Facility Layout deals with the selection of most appropriate and effective arrangements of departments in the open continual plane to allow greater working efficiency (Apple 1977, Deb et al. 2001a). Due complex and unstructured nature of FLs various approaches have been proposed by many researchers in the field. Irrespective of type of data, there is always an element of fuzziness or vagueness in it (Dweiri, 1999). The potential application of fuzzy set theory in the field of production management was illustrated effectively by Karwowski and Evans 1987. Deb et al. 2001a developed hybrid modeling for the management of material handling equipment selection planning while generating a manufacturing facility layout. The same

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authors (2001b) also proposed different projects of integrating FL and material handling equipment selection by using a knowledge base and optimization approach. Deb et al. (2002c) developed a decision model and algorithm for a material handling equipment selection routine under FL planning by using fuzzy MCDM methods. Taking a cue from the previous work, the present work focuses on integrating various Linguistic Variables to evaluate the FL selection procedure.

To strike a right balance between maximising resource utilisation and minimising overall cost incurred in production a right facilities layout selection is a very important issue in all the modern industrial organizations. Many potential FL attributes viz., Availability of Skilled workforce, Size & Shape of the Departments, Distance between facilities, Quality of production, various cost components involved, Lighting & Ventilation etc are considered for the selection of a right kind of FL for a particular type of production process. Likewise, various criteria were considered for facility site selection by Tompkins & White 1984, Spohrer & Kmak 1984 and Jarboe 1986. Similarly, many factors are to be considered to make an appropriate selection of a FL design. In the real-world, attributes so selected to help decision making regarding FL design selection can be categorised into: (1) **Subjective issues**(These factors have qualitative definition and based solely on an individual's(expert) perception and ratings)

such as availability of skilled workforce, size & shape of the departments, distance between facilities, quality of production etc;

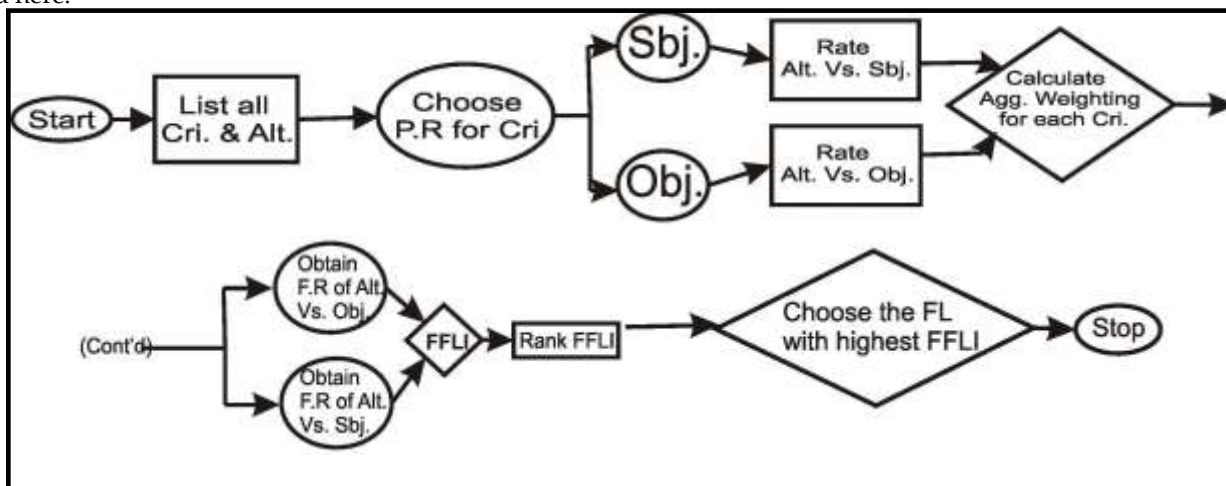
(2)**Objective issues** (These factors are defined in real quantitative terms) such as Investment cost, MH costs, Operating Costs, Improvement Costs etc. The basis for this can be found in Tompkins & White 1984. They introduced a method whereby the selection criteria regarding facilities were classified into 3 main categories: (1) Critical Factors, (2) Subjective Factors, (3) Objective Factors.

In reality, it is extremely difficult to precisely define the FL suitability measures, i.e. measures of subjective criteria for the decision makers to aid in their assessment. Besides, the evaluation data of the FL suitability under different criteria as well as the weight of the criteria will have to be expressed in a easily understandable language as linguistic terms, i.e. 'good', 'bad', 'very low', 'medium', 'high', 'poor', etc. That is why a Fuzzy Multi-Criteria Decision Making (FMCDM) method is needed to integrate various linguistic terms and corresponding weights to evaluate FL suitability and determine the selection of best layout. A measure called 'Fuzzy Facilities Layout Index (FFLI)' is proposed in this paper that handles fuzziness or vagueness inherent in the evaluation process and to provide a standard for selecting the most appropriate FL of the alternatives without losing sight of the importance of various criteria in FL selection process.

**Procedure of Facilities Layout Design Selection**

**Generation of a Model:**

A systematic approach to FL selection problem by using fuzzy set theory and hierarchical structure analysis like AHP is proposed here.



Cri. = Criteria, P.R = Preference Ratings, Alt. = Alternatives, Obj. = Objective Criteria, Sbj. = Subjective Criteria, F.R = Fuzzy Ratings, FFLI = Fuzzy Facilities Layout Index

**Figure 4: The Schematic Diagram of FL Selection Methodology**

**Proposed Methodology:**

Suppose there is a group of 'n' experts (DM<sub>1</sub>, DM<sub>2</sub>, . . . DM<sub>n</sub>) employs one or more rating sets to evaluate the

preferences. These decision makers are responsible for assessing the appropriateness of 'm' alternatives (FL<sub>1</sub>, FL<sub>2</sub>, . . . FL<sub>m</sub>) under each of 'k' criteria (C<sub>1</sub>, C<sub>2</sub>, . . . C<sub>k</sub>) as well as the importance of the criteria. Let S<sub>ij</sub> be the rating assigned to alternative FL<sub>i</sub> for the criterion C<sub>t</sub> by the decision-maker DM<sub>j</sub>. Let W<sub>tj</sub> be weight given to criterion C<sub>t</sub>, by the decision-maker DM<sub>j</sub>. Thus the committee has to first aggregate the ratings S<sub>ij</sub> of n decision makers for each alternative FL<sub>i</sub> versus each criterion C<sub>t</sub>, to form the rating S<sub>it</sub>. Each aggregated S<sub>it</sub>, i = 1, 2, . . . , m; t = 1, 2, . . . , k, can further be weighted by a weight W<sub>t</sub> according to the relative importance of the k criteria. Then, the final rating FFL<sub>i</sub> of alternative FL<sub>i</sub> can be obtained by aggregating S<sub>it</sub> and W<sub>t</sub> for

all selection criteria C<sub>t</sub>, t = 1, 2, . . . , k). Finally, rank the final rating FFL<sub>i</sub>, i = 1, 2, . . . , m, to obtain the most suitable layout.

The method of hierarchical structure analysis follows two distinct levels in this paper. In the first level, the fuzzy importance of decision criteria (e.g. size and shape of the departments, distance between facilities, quality of the product, lighting, ventilation and colours used etc.) are evaluated. In the second level, ratings (weights) are assigned to various FLs under each decision criterion. A hierarchical structure with four criteria and five alternatives are shown in the figure.4.

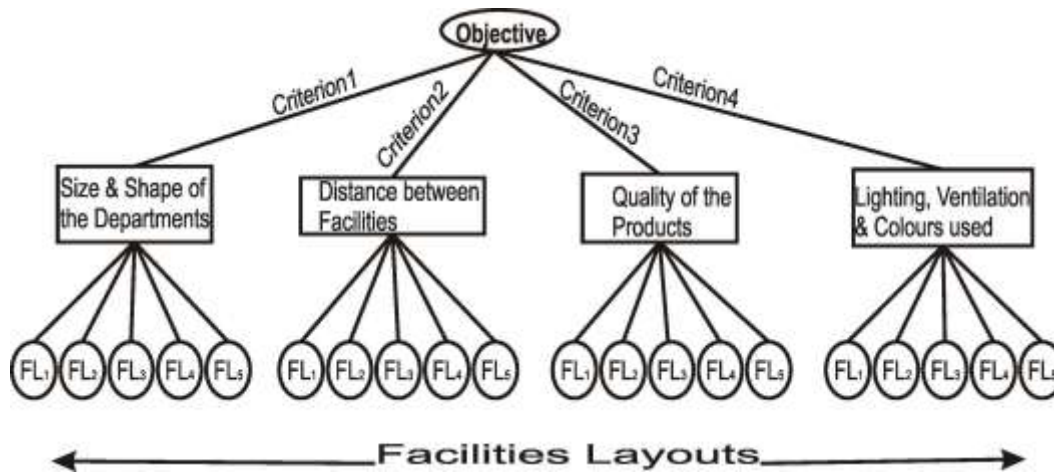


Figure 5: Hierarchical structure for 4 criteria and 5 alternatives.

(Subjective criteria - size and shape of the departments, distance between facilities, quality of the product, lighting, ventilation and colours used, etc. are characterized by linguistic assessments.

Objective Criteria – MH cost, Installation cost, Unit space cost, etc. are evaluated in monetary terms.)

**Rating System**

Two rating systems are used in present paper. Based on the practical needs and subjective assessment, the body of decision makers allocates numerical values to each one of them. The importance of weight of each criterion can be obtained by either directly assigning weight or indirectly using pair wise comparisons. The Linguistic Variable (LV)s weighting sets employed by the team of decision makers are W and S, where LV Set W = {F<sub>VL</sub>, F<sub>L</sub>, F<sub>M</sub>, F<sub>H</sub>, F<sub>VH</sub>} to evaluate the importance of the 'criteria'. Here F<sub>VL</sub> = Very Low, F<sub>L</sub> = Low, F<sub>M</sub> = Medium, F<sub>H</sub> = High, F<sub>VH</sub> = Very High. LV Set S = {F<sub>VP</sub>, F<sub>VPP</sub>, F<sub>P</sub>, F<sub>PF</sub>, F<sub>F</sub>, F<sub>FG</sub>, F<sub>G</sub>, F<sub>GVG</sub>, F<sub>VG</sub>}. Here F<sub>VP</sub> = Very Poor, F<sub>VPP</sub> = Somewhere in-between Very Poor and Poor, F<sub>P</sub> = Poor, F<sub>PF</sub> = Somewhere in-between Poor and Fair, F<sub>F</sub> = Fair, F<sub>FG</sub> = Somewhere in-between Fair

and Good, F<sub>G</sub> = Good, F<sub>GVG</sub> = Somewhere in-between Good and Very Good and F<sub>VG</sub> = Very Good. Using LV Set S the decision makers evaluate the suitability of alternatives versus subjective criteria. The above two sets can be simply represented by two separate rating scales (shown in the appendix).

**Objective vs. Subjective factors**

The compatibility between objective criteria (Cost) and subjective criteria (LV ratings) is to be ensured. For this, the cost component must be converted into dimensionless indices. 'Maximum rating is given to the alternative with the minimum cost'. Based on the principle stated above, we can write

$$FLRI_i = \left\{ \left[ \frac{1}{X_1} \oplus \frac{1}{X_2} \oplus \dots \oplus \frac{1}{X_m} \right] \otimes X_i \right\}^{-1} \tag{2}$$

where FLRLi=Facilities Layout Rating Index of the  $i^{th}$  alternative,  $X_i, i=1,2,\dots, m$  denote the cost component of alternative  $i$  (i.e. layout alternative  $i$ ).

**Overall fuzzy assessment**

As stated by Buckley 1984, there are many methods to aggregate fuzzy assessments. That is mean, median, max. value, min. value and mixed operator can be used. Since the most commonly used aggregation method

is the average operation, here we consider, the **mean operator**. In this paper overall fuzzy assessment is carried out using Liang & Wang 1991 who adopted Chen 1985 for final rating and ranking.

Let us assume that a decision-maker  $DM_i$  assigns a linguistic rating  $S_{ij} = (c_{ij}, a_{ij}, b_{ij}, d_{ij}), i = 1,2,\dots, m; t = 1,2,\dots, k - 1; j = 1,2,\dots, n$  to alternative  $FL_i$  for subjective criterion  $C_t$ . He also gives linguistic rating of weight to subjective criteria  $C_1, C_2, \dots, C_{k-1}$ , and objective criterion  $C_k$  as  $W_{ij} = (g_{ij}, e_{ij}, f_{ij}, h_{ij}), t = 1,2, \dots, k; j = 1,2, \dots, n$  respectively. Then,

$$S_{it} = \begin{cases} (1/n) \otimes (S_{it1} \oplus S_{it2} \oplus \dots \oplus S_{itm}) & i = 1, 2, \dots, m; t = 1, 2, \dots, k - 1 \\ (c_{ik}, a_{ik}, b_{ik}, d_{ik}) & i = 1, 2, \dots, m; t = k \end{cases} \quad (3)$$

and

$$W_t = (1/n) \otimes (W_{t1} \oplus W_{t2} \oplus \dots \oplus W_{tn}) \quad t = 1, 2, \dots, k \quad (4)$$

Thus,  $S_{it}, t = 1, 2, \dots, k-1$  is the mean of all the linguistic ratings of  $FL_i$  for subjective criterion  $C_t$ ,  $S_{ik}$  is the rating of  $FL_i$  versus Objective criterion  $C_k$  and  $W_t$  is the weight of criterion  $C_t$ .

Then,

$$S_{it} = (c_{it}, a_{it}, b_{it}, d_{it}) \quad t = 1, 2, \dots, k \quad (5)$$

and

$$W_t = (g_t, e_t, f_t, h_t) \quad t = 1, 2, \dots, k \quad (6)$$

Further,  $S_{it}$  and  $W_t$  are used to find out **Fuzzy Facilities Layout Index (FFLI<sub>i</sub>)** of the  $i^{th}$  alternative.

$$FFLI_i = (1/k) \otimes [(S_{i1} \otimes W_1) \oplus (S_{i2} \otimes W_2) \oplus \dots \oplus (S_{ik} \otimes W_k)] \quad (7)$$

The membership function of the fuzzy number  $FFLI_i$  is given as,

$$\mu_{FFLI_i}(x) = \begin{cases} -H_{i1} + [H_{i1}^2 + (x - Y_i)/T_{i1}] & Y_i \leq x \leq Q_i \\ 1 & Q_i \leq x \leq R_i \\ H_{i2} + [H_{i2}^2 + (x - Z_i)/T_{i2}] & R_i \leq x \leq Z_i \\ 0 & otherwise \end{cases} \quad \text{(using extension principle)} \quad (8)$$

for  $i = 1, 2, \dots, m$

where

$$T_{i1} = \sum_{t=1}^k (a_{it} - c_{it})(e_t - g_t)/k, \quad T_{i2} = \sum_{t=1}^k [c_{it}(e_t - g_t) + g_t(a_{it} - c_{it})]/k$$

$$U_{i1} = \sum_{t=1}^k (d_{it} - b_{it})(h_t - f_t)/k,$$

$$H_{i1} = T_{i2}/(2T_{i1}), \quad H_{i2} = -U_{i2}/(2U_{i1}) \quad \text{and}$$

$$Y_i = \sum_{t=1}^k c_{it}g_t/k, \quad Q_i = \sum_{t=1}^k a_{it}e_t/k, \quad R_i = \sum_{t=1}^k b_{it}f_t/k, \quad Z_i = \sum_{t=1}^k d_{it}h_t/k$$

Here,  $FFLI_i$  is expressed as  $FFLI_i = (Y_i, Q_i, R_i, Z_i; H_{i1}, T_{i1}; H_{i2}, U_{i1}) \quad i = 1, 2, \dots, m$  because it is not a trapezoidal fuzzy number.

This can be further simplified using the approximation formula,

$$FFLI_i \cong (Y_i, Q_i, R_i, Z_i)$$

This provides a trapezoidal fuzzy number that coincides with FFLi at the intervals  $[-\infty, Y_i], [Y_i, Q_i], [Q_i, R_i], [R_i, Z_i], [Z_i, \infty]$ .

### Ranking

Bortolan and Degani 1985, Buckley 1985, Chen 1985, Buckley and Chanas 1989, Campos and Gonzalez 1989, Gonzalez 1990, Kim and Park, 1990 proposed various methods for ranking fuzzy sets and fuzzy numbers. The question of ranking the alternatives is of utmost importance, because along with ranking, consistency in the result is to be obtained. Bortolan and Degani 1985, Kim & Park 1990, Liou & Wang 1992 are some of researchers who have done very appreciable work in the field of operations and ranking of fuzzy sets. A critical review of different

ranking methods was given by Bortolan and Degani 1985. Kim & Park 1990 developed a new ranking methodology and Liou & Wang 1992 proposed an Integral Value (IV) method of ranking fuzzy sets. Jain 1976 used the concept of maximising set that considers both maximum utility and grade of membership of the utilities. Though it is useful, a more sensitive rule is the one that takes into account a convex combination Max.Utility and Min.Utility using the index of optimism in the [0,1] interval as proposed by Kim & Park 1990.

### Maximizing Set and Minimizing Set

Let FFLi,  $i = 1, 2, \dots, m$  be the Fuzzy Facilities Layout Indices of 'm' alternatives, each with membership function obtained by equation (8). Define:

The maximizing set  $M = \{(x, \mu_M(x)) / x \in R\}$  with

$$\mu_M(x) = \begin{cases} [(x - x_1) / (x_2 - x_1)]^k & x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases}$$

and the minimizing set  $G = \{(x, \mu_G(x)) / x \in R\}$  with

$$\mu_G(x) = \begin{cases} [(x - x_2) / (x_1 - x_2)]^k & x_1 \leq x \leq x_2 \\ 0 & \text{otherwise} \end{cases}$$

where  $k > 0$ ,  $x_1 = \inf D$ ,  $x_2 = \sup D$ ,  $D = \bigcup_{i=1}^m D_i$ ,  $D_i = \{x \mid \mu_{F_i}(x) > 0\}$   $i = 1, 2, \dots, m$

The value of k can be changed to fit the decision-maker's preference in application. When  $k > 1$ , the decision-maker considers the maximizing set and the minimizing set m, convex function, i.e. as the value of x gets larger, the degree of preference of the decision maker increases rapidly. When  $k = 1$ , the decision-maker considers the maximizing set and the minimizing set as linear function, i.e. as the value of x gets larger, the degree of preference increases proportionally. When  $0 < k < 1$ , the decision-maker considers the maximizing set and the minimizing set as concave function, i.e. as the value of x gets larger, the

degree of preference increases more slowly than that of the other cases. Here,  $k = 1$  is applied.

### Rank Fuzzy Facilities Layout Indices (FFLI)

Ranking the Fuzzy Facilities Layout Indices is a necessary step for decision-makers to select a best layout alternative. A ranking method with the maximizing set and the minimizing set is proposed. We define the **right utility value**  $U_M(FFLI_i)$  and the **left utility value**  $U_G(FFLI_i)$  of each FFLi as

$$U_M(FFLI_i) = \sup_x (\mu_{FFLI_i}(x) \wedge \mu_M(x)) \tag{9}$$

and

$$U_G(FFLI_i) = \sup_x (\mu_{FFLI_i}(x) \wedge \mu_G(x)) \tag{10}$$

for  $i = 1, 2, \dots, m$ . Define the ranking value  $U_T(FFLI_i)$  of the FFLi as

$$FFLI_i = (Y_i, Q_i, R_i, Z_i; H_{i1}, T_{i1}; H_{i2}, U_{i1})$$

each with the membership function given by equation (8),

$$\text{i.e. } U_T(FFLI_i) = [U_M(FFLI_i) + 1 - U_G(FFLI_i)]/2 \quad (11)$$

for  $i = 1, 2, \dots, m$ .

By using equation (8), (9), (10) and (11), the ranking values  $U_T(FFLI_i)$  of Fuzzy Facilities Layout Indices  $FFLI_i$  can be obtained:

$$U_T(FFLI_i) = \left[ H_{i2} - \left( H_{i2}^2 + (x_R - Z_i)/U_{i1} \right)^{1/2} + 1 + H_{i1} - \left( H_{i1}^2 + (x_L - Y_i)/T_{i1} \right)^{1/2} \right] / 2 \quad (12)$$

for  $i = 1, 2, \dots, m$ .

For simplicity, the ranking value  $U_T(FFLI_i)$  of fuzzy facility layout index can be approximated by the ranking value of trapezoidal fuzzy number  $(Y_i, Q_i, R_i, Z_i)$  which the equation is :

$$U_T(FFLI_i) \cong \left[ \left( (Z_i - x_1) / ((x_2 - x_1) - (R_i - Z_i)) + 1 - (x_2 - Y_i) / (x_2 - x_1) + (Q_i - Y_i) \right) \right] / 2 \quad (13)$$

for  $i = 1, 2, \dots, m$ .

Let  $FFLI_i$  and  $FFLI_j$  be the fuzzy facilities layout indices of alternatives  $i$  and  $j$ .

Let us define that  $FFLI_i > FFLI_j$

if and only if,  $U_T(FFLI_i) > U_T(FFLI_j)$  or  $U_T(FFLI_i) = U_T(FFLI_j)$

but  $(Q_i + R_i) > (Q_j + R_j)$ ;

and that,  $FFLI_i = FFLI_j$

if and only if,  $U_T(FFLI_i) > U_T(FFLI_j)$

and  $(Q_i + R_i) = (Q_j + R_j)$ .

By equation (12) or (13) the ranking values of the fuzzy facilities layout indices of 'm' facilities layouts can be easily calculated. The best facility layout design can be selected by the decision makers' committee easily based on this ranking.

### Summary of the procedure

- A group of experts is constituted and to fix the alternative layouts and criteria to be considered.
- Relevant choice ratings for each facilities layout are identified.
- The criteria chosen are to be segregated as objective and subjective criteria.
- Choice ratings for appropriateness of alternatives vs. criteria are determined.
- The results are tabulated and aggregated weighting  $W_t$  is obtained.
- Obtain Aggregate fuzzy ratings  $S_{it}$  of alternative  $A_i$  for criterion  $C_t$ .
- Tabulate the costs with different alternative FLDs and then assign FLR<sub>i</sub>.
- Fuzzy Facilities Layout Indices  $FFLI_i$  for all the alternatives are calculated after obtaining aggregated  $S_{it}$  and  $W_t$  with reference to each criterion.
- Fuzzy Facilities Layout Index  $FFLI_i$  are to be ranked after calculating  $(U_T(FFLI_i))$  associated with each of the alternatives.
- Select the FL with the highest rank value.

### A Case Study

A CNC machine tool industry manufacturing machine tool components in Bangalore, India, was selected to apply this procedure and ascertain the facts involved. In this section a FLD selection problem was solved to demonstrate the computational process of this fuzzy FL selection algorithm described earlier.

**Step 1 :** This multi-product manufacturing firm needs to choose a suitable layout for the current production system. 5 FLDs  $FL_1, FL_2, FL_3, FL_4$  and  $FL_5$  were taken up. A group consisting of four experts  $DM_1, DM_2, DM_3$  and  $DM_4$  was formed. Five selection criteria were considered:

1. Size and Shape of the Departments ( $C_1$ ) ;
2. Distance between Facilities ( $C_2$ ) ;
3. Quality of the Products ( $C_3$ ) ;
4. Lighting, Ventilation and Identification Colours used ( $C_4$ ) ;
5. Total Cost ( $C_5$ ).

**Step 2 :** Employ the linguistic weighting set  $W = \{F_{VL}, F_L, F_M, F_H, F_{VH}\}$ .

**Step 3 :** The criteria are classified into two groups as shown in Table 1.

Subjective Criteria	Objective Criteria
<ul style="list-style-type: none"> <li>• Size and Shape of the Departments (<math>C_1</math>) ;</li> <li>• Distance between Facilities (<math>C_2</math>) ;</li> <li>• Quality of the Products (<math>C_3</math>) ;</li> <li>• Lighting, Ventilation and Identification Colours used (<math>C_4</math>).</li> </ul>	* Total Investment Cost ( $C_5$ ).

**Table 1: Criteria for FLDs**

**Step 4 :** Use the LV set  $S = \{F_{VP}, F_{VPP}, F_P, F_{PF}, F_F, F_{FG}, F_G, F_{GVG}, F_{VG}\}$  to evaluate the alternatives under each of the subjective criteria. The rating of the objective criteria is given by  $FLR_i, i = 1, 2, 3 \dots$  as shown in Eq. 2.

**Step 5 :** The fuzzy linguistic weights allotted to the five criteria( $C_1 - C_5$ ) by the four experts is presented in Table 2.

Criteria	Decision –makers			
	DM <sub>1</sub>	DM <sub>2</sub>	DM <sub>3</sub>	DM <sub>4</sub>
C <sub>1</sub>	F <sub>M</sub>	F <sub>VH</sub>	F <sub>M</sub>	F <sub>H</sub>
C <sub>2</sub>	F <sub>H</sub>	F <sub>VH</sub>	F <sub>VH</sub>	F <sub>M</sub>
C <sub>3</sub>	F <sub>VH</sub>	F <sub>H</sub>	F <sub>H</sub>	F <sub>VH</sub>
C <sub>4</sub>	F <sub>VH</sub>	F <sub>H</sub>	F <sub>VH</sub>	F <sub>VH</sub>
C <sub>5</sub>	F <sub>VL</sub>	F <sub>L</sub>	F <sub>H</sub>	F <sub>VH</sub>

**Table 2: The criteria ratings using LV set ‘W’**

**Step 6 :** By using eqn. 4, the aggregated weighting  $W_t$  of the decision-making committee can be obtained.

$$W_1 = (0.325, 0.675, 0.675, 0.900) \quad W_2 = (0.450, 0.800, 0.800, 0.950)$$

$$W_3 = (0.550, 0.850, 0.850, 1.000) \quad W_4 = (0.575, 0.925, 0.925, 1.000)$$

$$W_5 = (0.275, 0.475, 0.475, 0.650)$$

**Step 7 :** Enumerate the decision makers’ assessment of subjective criteria for FLDs.

Alternative Layouts	Decision –makers			
	DM <sub>1</sub>	DM <sub>2</sub>	DM <sub>3</sub>	DM <sub>4</sub>
FL <sub>1</sub>	F <sub>VPP</sub>	F <sub>F</sub>	F <sub>PF</sub>	F <sub>FG</sub>
FL <sub>2</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>F</sub>	F <sub>G</sub>
FL <sub>3</sub>	F <sub>PF</sub>	F <sub>GVG</sub>	F <sub>PF</sub>	F <sub>FG</sub>
FL <sub>4</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>G</sub>	F <sub>F</sub>
FL <sub>5</sub>	F <sub>GVG</sub>	F <sub>VG</sub>	F <sub>FG</sub>	F <sub>P</sub>

**Table 3: Decision makers’ subjective assessment of alternatives under C<sub>1</sub>**

Alternative Layouts	Decision –makers			
	DM <sub>1</sub>	DM <sub>2</sub>	DM <sub>3</sub>	DM <sub>4</sub>
FL <sub>1</sub>	F <sub>PF</sub>	F <sub>F</sub>	F <sub>P</sub>	F <sub>F</sub>
FL <sub>2</sub>	F <sub>G</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>VG</sub>
FL <sub>3</sub>	F <sub>F</sub>	F <sub>FG</sub>	F <sub>P</sub>	F <sub>F</sub>
FL <sub>4</sub>	F <sub>GVG</sub>	F <sub>G</sub>	F <sub>FG</sub>	F <sub>PF</sub>
FL <sub>5</sub>	F <sub>FG</sub>	F <sub>G</sub>	F <sub>PF</sub>	F <sub>FG</sub>

**Table 4: Decision makers’ subjective assessment of alternatives under C<sub>2</sub>**

Alternative Layouts	Decision –makers			
	DM <sub>1</sub>	DM <sub>2</sub>	DM <sub>3</sub>	DM <sub>4</sub>
FL <sub>1</sub>	F <sub>G</sub>	F <sub>G</sub>	F <sub>G</sub>	F <sub>FG</sub>
FL <sub>2</sub>	F <sub>PF</sub>	F <sub>F</sub>	F <sub>GVG</sub>	F <sub>G</sub>
FL <sub>3</sub>	F <sub>FG</sub>	F <sub>P</sub>	F <sub>FG</sub>	F <sub>VG</sub>
FL <sub>4</sub>	F <sub>GVG</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>FG</sub>
FL <sub>5</sub>	F <sub>G</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>F</sub>

Table 5: Decision makers’ subjective assessment of alternatives under C<sub>3</sub>

Alternative Layouts	Decision –makers			
	DM <sub>1</sub>	DM <sub>2</sub>	DM <sub>3</sub>	DM <sub>4</sub>
FL <sub>1</sub>	F <sub>PF</sub>	F <sub>F</sub>	F <sub>GVG</sub>	F <sub>P</sub>
FL <sub>2</sub>	F <sub>FG</sub>	F <sub>P</sub>	F <sub>VPP</sub>	F <sub>G</sub>
FL <sub>3</sub>	F <sub>G</sub>	F <sub>FG</sub>	F <sub>PF</sub>	F <sub>F</sub>
FL <sub>4</sub>	F <sub>F</sub>	F <sub>VG</sub>	F <sub>FG</sub>	F <sub>PF</sub>
FL <sub>5</sub>	F <sub>VPP</sub>	F <sub>F</sub>	F <sub>G</sub>	F <sub>FG</sub>

Table 6: Decision makers’ subjective assessment of alternatives under C<sub>4</sub>

Alternative Layouts	Design Cost (\$ X 10 <sup>6</sup> )	MH Cost (\$ X 10 <sup>6</sup> )	Implementation Cost (\$ X 10 <sup>6</sup> )	Cost of Unit Space Cost(\$ X 10 <sup>6</sup> )
FL <sub>1</sub>	(2,3,4,5)	(8,10,10,12)	(9,9,9,9)	(1,2,2,3)
FL <sub>2</sub>	(8,9,9,12)	(9,9,9,9)	(6,8,8,10)	(1,1,1,1)
FL <sub>3</sub>	(1,3,3,5)	(3,4,4,5)	(3,3,3,3)	(4,6,6,8)
FL <sub>4</sub>	(10,12,12,14)	(14,14,14,14)	(12,14,14,15)	(6,6,6,6)
FL <sub>5</sub>	(6,8,8,9)	(5,8,8,10)	(7,9,9,11)	(2,3,3,4)

(Approximate values between the Min. and Max. Cost values under each head in millions of dollars are listed above)

(Cont’d Table 7)

Total Cost T <sub>i</sub> X \$10 <sup>6</sup>	T <sub>i</sub> X G $G = 10^{-2} \otimes [T_1^{-1} \oplus T_2^{-1} \oplus T_3^{-1} \oplus T_4^{-1}]$	FLRI <sub>i</sub> = S <sub>i 5</sub> i=1,2,...5
(20, 24, 25, 29)	(3.26, 4.73, 4.97, 7.44)	(0.134, 0.201, 0.211, 0.307)
(24, 27, 27, 32)	(3.92, 5.32, 5.37, 8.20)	(0.122, 0.186, 0.188, 0.255)
(11, 16, 16, 21)	(1.80, 3.15, 3.18, 5.38)	(0.186, 0.314, 0.317, 0.556)
(42, 46, 46, 49)	(6.85, 9.06, 9.14, 12.56)	(0.080, 0.109, 0.110, 0.146)
(20, 28, 28, 34)	(3.26, 5.52, 5.56, 8.72)	(0.115, 0.180, 0.181, 0.307)

Table 7: Costs of alternative FLDs and FLRI ratings

Step 8 : By using Eq. 3, S<sub>it</sub>, i = 1, 2, 3, 4, 5; t = 1, 2, 3, 4 are obtained.

$$S_{11} = (0.150, 0.325, 0.525, 0.7875) \quad S_{21} = (0.300, 0.525, 0.650, 0.925)$$

$$S_{31} = (0.225, 0.475, 0.700, 0.900) \quad S_{41} = (0.300, 0.525, 0.650, 0.925)$$

$$S_{51} = (0.450, 0.625, 0.750, 0.875)$$

$$S_{12} = (0.150, 0.375, 0.425, 0.775) \quad S_{22} = (0.450, 0.650, 0.775, 0.950)$$

$$S_{32} = (0.225, 0.425, 0.500, 0.825) \quad S_{42} = (0.375, 0.600, 0.775, 0.950)$$

$$S_{52} = (0.300, 0.400, 0.725, 0.950)$$



$$S_{13} = (0.525, 0.725, 0.800, 1.000) \quad S_{23} = (0.375, 0.600, 0.700, 0.925)$$

$$S_{33} = (0.375, 0.550, 0.700, 0.875) \quad S_{43} = (0.300, 0.525, 0.775, 0.950)$$

$$S_{53} = (0.300, 0.525, 0.650, 0.925)$$

$$S_{14} = (0.225, 0.450, 0.550, 0.800) \quad S_{24} = (0.225, 0.375, 0.525, 0.7375)$$

$$S_{34} = (0.300, 0.525, 0.650, 0.925) \quad S_{44} = (0.375, 0.575, 0.700, 0.925)$$

$$S_{54} = (0.300, 0.450, 0.600, 0.8375)$$

**Step 9:** The experts' committee assesses the fuzzy or non-fuzzy costs associated with each of the alternative FLDs and the corresponding FLRI<sub>i</sub> are listed as S<sub>i5</sub> values.

$$S_{15} = (0.134, 0.201, 0.211, 0.307) \quad S_{25} = (0.122, 0.186, 0.188, 0.255)$$

$$S_{35} = (0.186, 0.314, 0.317, 0.556) \quad S_{45} = (0.080, 0.109, 0.110, 0.146)$$

$$S_{55} = (0.115, 0.314, 0.317, 0.556)$$

**Step 10:** The FFLI<sub>i</sub> are obtained by aggregating S<sub>it</sub> and W<sub>t</sub> of all the criteria. (Eq. 5,6,7,8)

Alternative Layouts	Fuzzy Facilities Layout Indices (FFLI) $FFLI_i = (Y_i, Q_i, R_i, Z_i; H_{i1}, T_{i1}; H_{i2}, U_{i1})$
FL <sub>1</sub>	(0.144, 0.329, 0.397, 0.689; 1.343, 0.058; 4.622, 0.035)
FL <sub>2</sub>	(0.134, 0.364, 0.446, 0.713; 1.543, 0.056; 4.964, 0.029)
FL <sub>3</sub>	(0.121, 0.353, 0.444, 0.751; 1.343, 0.063; 4.709, 0.036)
FL <sub>4</sub>	(0.134, 0.373, 0.483, 0.741; 1.487, 0.060; 4.211, 0.039)
FL <sub>5</sub>	(0.130, 0.351, 0.469, 0.763; 1.654, 0.051; 5.015, 0.033)

**Table 8: Fuzzy Facilities Layout Indices of five alternative layouts**

**Step 11 :** The ranking of each of the FLD's FFLI<sub>i</sub> is carried out using Eq.(9-13). The results are tabulated in Table 9. The alternative with highest FFLI<sub>i</sub> will be the recommended FLD for the production process.

Alternative Layouts	FFLI <sub>i</sub>	Rank
FL <sub>1</sub>	0.3324	5
FL <sub>2</sub>	0.3933	3
FL <sub>3</sub>	0.3561	4
FL <sub>4</sub>	<u>0.4251</u>	<u>1</u>
FL <sub>5</sub>	0.3987	2

**Table 9: Ranking of Alternatives**

**(FL<sub>4</sub> is the alternative with highest rank and hence is selected.)**

### DISCUSSION AND CONCLUSION:

This paper explores the possibility of a FLD selection process under fuzzy environment. It also considers the presence of many conflicting and contradicting factors influencing FLD by presenting an algorithm based on a MCDM technique.

The conventional approaches are deterministic in nature. Also, any random process tends to be less effective in handling imprecise or vague real-world situations. The present methodology considers both. It effectively handles vague assessments expressed as LVs and takes into account multi-factors influencing FLDs. Perhaps, the fuzzy numbers and LVs are used to evaluate the objective and

subjective factors in such a manner that the views of the entire group of experts can be expressed clearly. The uniqueness of this approach lies in the fact that using this method a real situation can be modeled as it is. This method can be computerised by which just by conducting fuzzy linguistic assessments as well as fuzzy or non-fuzzy objective assessment, the decision makers can obtain ranking of the alternatives directly.

As FL<sub>4</sub> has highest FFLI<sub>i</sub>, it can be selected. This selection is appropriate, though it is found to incur more in terms of total investment. The benefits from this FLD on the issues of Size and Shape of the Departments, Distance between Facilities, Quality of the Products manufactured, Lighting, Ventilation and Identification colours used

outweigh the cost as it is evident from the ratings given by the experts.

many other areas of management decision making problems involving MCDM environment although, this algorithm highlights a FLD selection.

This methodology can also be applied to problems such as project management, machinery selection and

## Appendix

### Fuzzy Set Theory

It was Lofti A.Zadeh who propounded the fuzzy set theory in his seminal paper (Zadeh, 1965) that exhibits immense potential for effective solving of the uncertainty in the real world problems. Fuzzy sets unlike crisp sets support a flexible sense of membership of elements to a set. i.e. under fuzzy set theory many degrees of membership (between 0 and 1) are allowed. Thus a formal mapping is written as

$$\mu_{\tilde{A}}(x) : X \rightarrow [0,1] \text{ where } \mu_{\tilde{A}}(x) \text{ is a membership function associated with a fuzzy set } \tilde{A}$$

such that the function maps every element of the universe of discourse X(or the reference set) to the interval [0,1]. It has become a modeling language to approximate situations in which fuzzy phenomenon and criteria exist. Larger the  $\mu_{\tilde{A}}(x)$  value, stronger the grade of membership for x in A.

### Fuzzy numbers

A fuzzy number B is a special subset of real numbers (Jain 1976, Dubois and Prade 1978). Its membership function  $\mu_B$  is a continuous mapping from X to a closed interval [0, 1], which has the following characteristics:

- (1)  $\mu_B(x) = 0$  for all  $x \in (-\infty, a) \cup (d, \infty)$ ;
- (2)  $\mu_B$  is strictly increasing in [a, b] and strictly decreasing in [c,d];
- (3)  $\mu_B(x) = 1$ , for all  $x \in (b, c)$ .

It can be  $a = -\infty$  or  $a = b$  or  $b = c$  or  $c = d$  or  $d = \infty$ . This kind of fuzzy numbers that employ straight line segments for  $\mu_B(x)$  in [a, b] and [c, d] are called trapezoidal fuzzy numbers.

A fuzzy number A in X is a trapezoidal fuzzy number if its membership function  $\mu_A: X \rightarrow [0, 1]$  is

$$\mu_A(x) = \begin{cases} (x-r)/(p-r) & r \leq x \leq p \\ 1 & p \leq x \leq q \\ (x-s)/(q-s) & q \leq x \leq s \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

with  $r \leq p \leq q \leq s$ . The trapezoidal fuzzy number can be denoted by (r, p, q, s) as shown by Eq.1.

### Linguistic Variable (LV)s :

LVs are adopted to deal with uncertain, imprecise and vague situations. As explained by Zadeh 1975 – 76 these variables are defined reasonably in conventional linguistic quantitative expressions expressing too complex or too ill-  
 $\mu_w(x)$

defined situations. Thus it is a variable whose values are words or sentences in natural or artificial language. An example is shown in Figure 1.

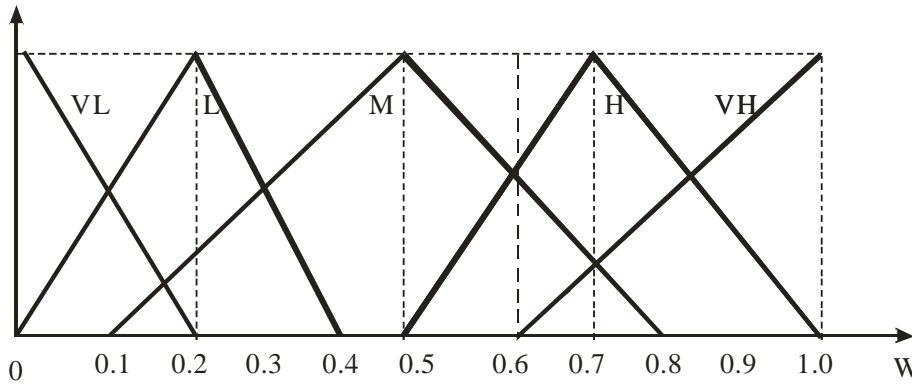
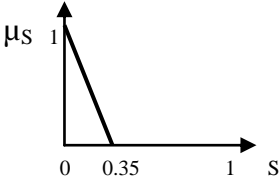
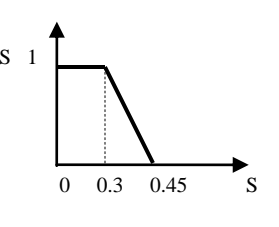
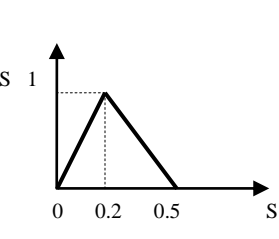
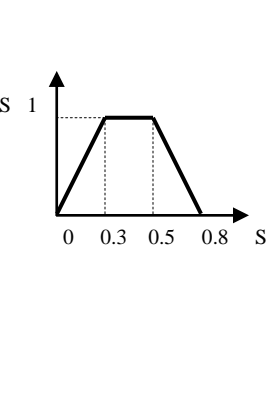
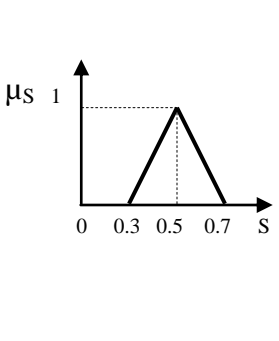


Figure 1 : Membership Function for LVs ( $F_{VL} = \text{very low} : (0, 0, 0, 0.2)$ ,  $F_L = \text{low} : (0, 0.2, 0.2, 0.4)$ ,  $F_M = \text{medium} : (0.1, 0.5, 0.5, 0.8)$ ,  $F_H = \text{high} : (0.5, 0.7, 0.7, 1)$ ,  $F_{VH} = \text{very high} : (0.6, 1, 1, 1)$ )

The membership function values in LV set 'W' :

Membership Function	Pictorial Representation
$F_{VL} : (0, 0, 0, 0.2)$ $\mu_w(x) = 1 - 5x \quad 0 \leq x \leq 0.2$	
$F_L : (0, 0.2, 0.2, 0.4)$ $\mu_w(x) = \begin{cases} 5x & 0 \leq x \leq 0.2 \\ 2 - 5x & 0.2 \leq x \leq 0.4 \end{cases}$	
$F_M : (0.1, 0.5, 0.5, 0.8)$ $\mu_w(x) = \begin{cases} \frac{5x}{2} - \frac{5}{2} & 0.1 \leq x \leq 0.5 \\ \frac{8}{3} - \frac{10x}{3} & 0.5 \leq x \leq 0.8 \end{cases}$	
$F_H : (0.5, 0.7, 0.7, 1)$ $\mu_w(x) = \begin{cases} \frac{7x}{5} - \frac{7}{10} & 0.5 \leq x \leq 0.7 \\ \frac{10}{3} - \frac{10x}{3} & 0.7 \leq x \leq 1 \end{cases}$	
$F_{VH} : (0.6, 1, 1, 1)$ $\mu_w(x) = \begin{cases} \frac{5x}{2} - \frac{3}{2} & 0.6 \leq x \leq 1 \end{cases}$	

**The membership function value of LV Set 'S' :**

Membership Function	Pictorial Representation
$F_{VFP} : (0, 0, 0, 0.35)$ $\mu_s(x) = \begin{cases} 1 - \frac{20x}{7} & 0 \leq x \leq 0.35 \end{cases}$	
$F_{VFP} : (0, 0, 0.3, 0.45)$ $\mu_s(x) = \begin{cases} 1 & 0 \leq x \leq 0.3 \\ 2 - \frac{20x}{3} & 0.3 \leq x \leq 0.45 \end{cases}$	
$F_P : (0, 0.2, 0.2, 0.5)$ $\mu_s(x) = \begin{cases} 5x & 0 \leq x \leq 0.2 \\ \frac{5}{3} - \frac{10x}{3} & 0.2 \leq x \leq 0.5 \end{cases}$	
$F_{PF} : (0, 0.3, 0.5, 0.8)$ $\mu_s(x) = \begin{cases} \frac{10x}{3} & 0 \leq x \leq 0.3 \\ 1 & 0.3 \leq x \leq 0.5 \\ \frac{8}{3} - \frac{10x}{3} & 0.5 \leq x \leq 0.8 \end{cases}$	
$F_F : (0.3, 0.5, 0.5, 0.7)$ $\mu_s(x) = \begin{cases} 5x - \frac{3}{2} & 0.3 \leq x \leq 0.5 \\ \frac{5}{2} - 5x & 0.5 \leq x \leq 0.7 \end{cases}$	

$F_{FG} : (0.3, 0.5, 0.8, 1)$ $\mu_s(x) = \begin{cases} 5x - \frac{3}{2} & 0.3 \leq x \leq 0.5 \\ 1 & 0.5 \leq x \leq 0.8 \\ 5 - 5x & 0.8 \leq x \leq 1 \end{cases}$	
$F_G : (0.6, 0.8, 0.8, 1)$ $\mu_s(x) = \begin{cases} 5x - 3 & 0.6 \leq x \leq 0.8 \\ 5 - 5x & 0.8 \leq x \leq 1 \end{cases}$	
$F_{GVG} : (0.6, 0.8, 1, 1)$ $\mu_s(x) = \begin{cases} 5x - 3 & 0.6 \leq x \leq 0.8 \\ 1 & 0.8 \leq x \leq 1 \end{cases}$	
$F_V : (0.9, 1, 1, 1)$ $\mu_s(x) = \begin{cases} 10x - 9 & 0.9 \leq x \leq 1 \end{cases}$	

## 8. REFERENCES :

- Apple, J.M., 1977, Plant Layout and Material Handling (New York:Wiley).
- Bortolan.G. and Degani.R, 1985, A review of some methods of ranking fuzzy subsets. Fuzzy Sets and Systems, 15, 1 - 19.
- Buckley.J.J., 1984, The multiple judge, Multiple criteria ranking problem : A fuzzy set approach, Fuzzy Sets and Systems, 13, 25 - 37.
- Beckley.J.J., 1985, Ranking alternatives using fuzzy numbers, Fuzzy Sets and Systems, 15, 21 - 31.
- Beckley.J.J. and Chanas.S., 1989, A fast method of ranking alternative using fuzzy numbers: Fuzzy Sets and Systems, 30, 337 - 338.
- Campos Ibanez.L.M and Gonzalez Munoz.A., 1989, A subjective approach for ranking fuzzy numbers, Fuzzy Sets and Systems, 29, 145 - 153.
- Chen.S.H., 1985, Rankign fuzzy numbers with maximizing set and minimizing set. Fuzzy Sets and Systems, 17, 113 - 129.
- Dahlberg.M.D. and May J.H., 1980, Linear programming for siting of energy facilities, Journal of Energy Engineering, 5 - 14.
- Deb, S.K., Bhattacharya, B. & Sorkhel, S.K., 2001a, Management of machine layouts and material handling system selection using hybrid approach. Proceedings of International Conference on Logistic and Supply Chain Management, PSG Tech, Coimbatore, India, pp. 50-55.
- Deb, S.K., Bhattacharya, B. & Sorkhel, S.K., 2001b, Facility Layout and Material Handling equipment selection planning using hybrid methodology. International Journal of Industrial Engineering, 10, (3).
- Deb, S.K., Bhattacharya, B. & Sorkhel, S.K., 2002c, Material Handling Equipment Selection by Fuzzy Multi-Criteria Decision Making methods(Berlin:Springer), pp, 99-105.
- Deb, S.K., Bhattacharya, B., 2003, Facilities layout planning based on Fuzzy multiple criteria decision making methodology, IJPR, 2003, 41(18), 4487-4504.
- Dubois.D, and Prade H., 1978, Operations on fuzzy numbers. International journal on Systems Science, 9, 613 - 626.
- Dweiri, F & Meier, F. A, 1996, Application of fuzzy decision making in Facilities Layout Planning, IJPR, 34, 1431-1450.
- Dweiri, F., 1999, Fuzzy development of crisp activity relationship charts for facilities layout, International Journal of Computer and Industrial Engineering, 36, 1-16.
- George J.Klier, Tina A.Folger, Fuzzy Sets, Uncertainty and Information, Prentice-Hall of India Private Ltd., 2004
- Gin-Shun Liang and Mao-Jiun J. Wang, A fuzzy multi-criteria decision making method for facility site selection, IJPR, 1991, Vol.29, No.11, 2313 - 30.
- Gonzalez Munoz.A., 1990, A study of the ranking function approach through mean values, Fuzzy Sets and Systems, 35, 29 - 41.
- Karsak, E.E. and Tolga, E., 2001, Fuzzy multi-criteria decision making procedure evaluating advanced manufacturing system investments. International Journal Production Economics, 69, 49-64.
- Karsak, E.E., 2002, Distance-based fuzzy MCDM approach for evaluating flexible manufacturing system alternatives, IJPR, Vol.40, N0.13, 3167-3181.

21. Karwowski, W & Evans, G.W., 1987, A layout design heuristic employing theory of fuzzy set. *IJPR*, 25, 1431-1450.
22. Kim.K. and Park.K.S., 1990, Ranking fuzzy numbers with index of optimism, *Fuzzy Sets and Systems*, 35, 143 - 150.
23. Liang. G and Wang.M.J, 1991, A fuzzy multi-criteria decision-making method for facility site selection, *IJPR*, Vol.29, No.11, 2313-2330.
24. Rajashekar, S, Vijayalakshmi Pai, G.A, *Neural Networks, Fuzzy Logic and Genetic Algorithms, Synthesis and Applications*, Prentice-Hall of India Private Ltd., 2005
25. Spohrer.G.A. and Kmak.T.R., 1984, Qualitative analysis used in evaluating alternative plant location scenarios, *Industrial Engineering*, August, 52 - 56.
26. Tompkins.J.A. and White.J.A., 1984, *Facilities Planning* (New York: John Wiley & Sons),
27. Zadeh.L.A., 1965, *Fuzzy Sets*, *Information and Control*, 8, 338 - 353.
28. Zadeh.L.A., 1975/76, The concept of linguistic variable and its application to approximate reasoning, Part I, II and III, *Information Science*, 8, 199 - 249, 301 - 357 : 9, 43 - 58.
29. Zeki Ayag, An analytic hierarchy process based simulation model for implementation analysis of computer aided systems, *IJPR*, 2002, Vol.40, No.13, 3053 - 73.